

Your Signature _____

Instructions:

- (a) Please write your name on every page.*
- (b) Maximum time is 2 hours. Please stop writing when you are asked to do so.*
- (c) You may use any result proved in class. When using a result, please state the result precisely.*
- (d) Do not use any results from homework assignments.*
- (e) Provide adequate justification for answers to the questions below.*

Score

1.	(15)	
2.	(15)	
3.	(20)	
Total.	(50)	

1. (a) Evaluate the Riemann integral:

$$\int_0^1 \sin(x) \cos(x) dx$$

(b) Show that the answer from above is the same as :-

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \sin \frac{i}{n} \cos \frac{i}{n}$$

2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ be given by

$$f(x) = x_1^2 - x_1 x_2 \quad \text{and} \quad g(x) = \begin{cases} 1 & x_1^2 \leq x_2 \text{ or } x_2 < 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let $u \in \mathbb{R}^2$. Find $D_u f(0)$ and $D_u g(0)$ (if they exist).

3. Decide whether the following statements are true or false:

(a) Let $g : [a, b] \rightarrow \mathbb{R}$. If g is Riemann integrable on $[c, b]$ for all $c \in (a, b)$ then g is Riemann integrable on $[a, b]$.

(b) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, be given by

$$f(x) = \begin{cases} \frac{x_1^2 x_2}{\sqrt{x_1^4 + x_2^4}} & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Then,

(i) f has all directional derivatives at 0

(ii) f is continuous at 0

(iii) f is differentiable at 0